Since this was a talk, the slides were just a helper to support my presentation.

For more information and a deeper understanding please check the literature on my homepage.

www.igi.tugraz.at/helmut/

Helmut Hauser, Aug.19th 2004

Survey of Adaptive Control

by Helmut Hauser

TUG

Structure of the Presentation

- Introduction and Review to Control Theory

- Different Approaches (linear)
  - Gain Scheduling
  - Model Adaptive Control
  - Self Tuning Regulators
  - Dual Control

- Nonlinear Approaches

- Conclusions

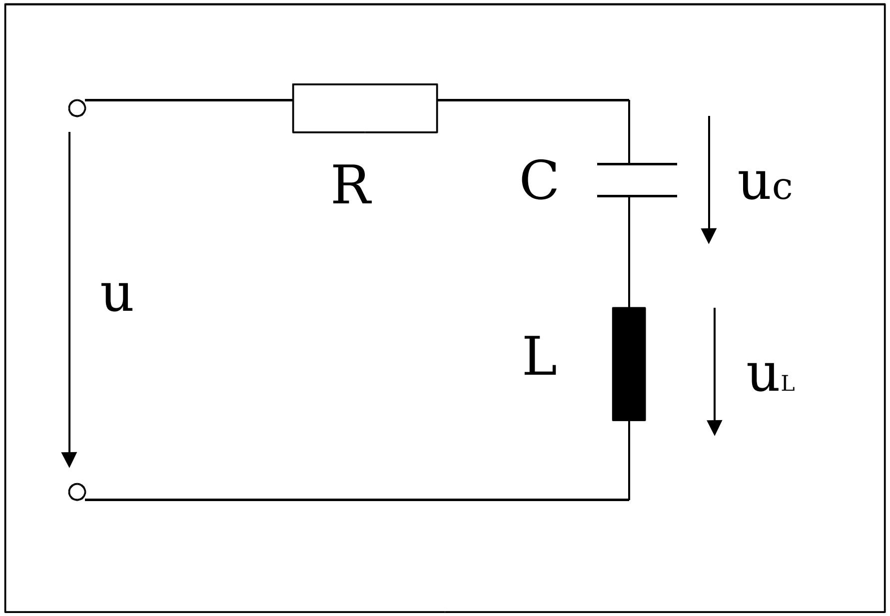
Control Theory
Formulation of the problem

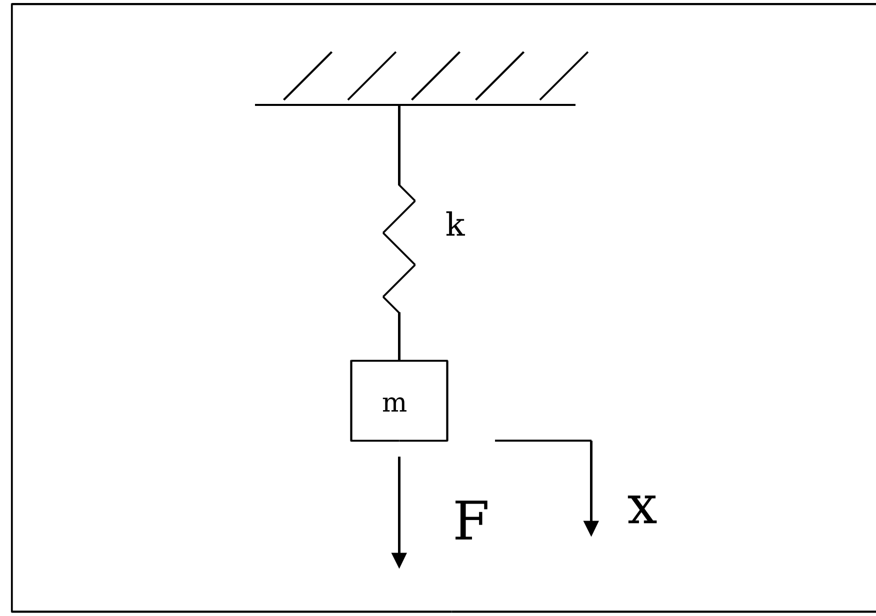
What is Control Theory ?:

- treating with dynamical systems
- mathematical formulation
- analysing their properties

-> Trying to influence somehow the behaviour and properties by another dynamical system

Dynamical Systems – Mathematical Formulation





Description by Kirchhoff´s law:

$$\sum u = 0$$

$$i_C = C \cdot \frac{du}{dt} \qquad u_L = L \cdot \frac{di}{dt}$$

Description by Newton:

$$\sum F = \sum m \cdot \frac{dx^2}{d^2t} = 0$$

$$F_{spring} = k \cdot x$$

Mathematical Formulation of the problems

Differential equations of order n

decoupling

Laplacetransformation

$$\dot{x} = A \cdot x + B \cdot u$$
$$y = C \cdot x$$

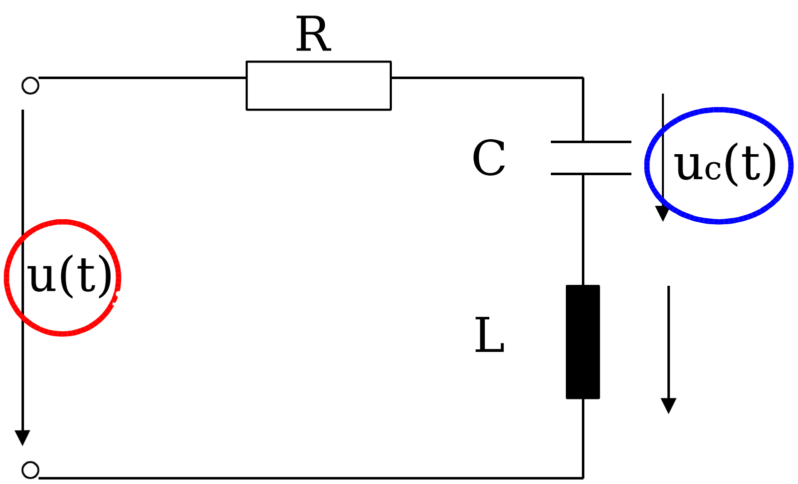
State Space Presentation

(N differential equations of order 1)

$$y(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots b_n s^n}{1 + a_1 s + a_2 s^2 + \ldots + a_m s^m} \cdot u(s)$$

Transfer Function

Example from before



$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot u$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\dot{x} = A \cdot x + B \cdot u$$
$$y = C \cdot x$$

$$G(s) = C \cdot (s \cdot I - A)^{-1} \cdot B + D$$

Properties of Dynamical Systems

- **Stability**
- Impulse Response
- Observability
- Controllability
- Causality
- much more...

real{eigvalues(A)}<1

State Space Representation

$$\dot{x} = A \cdot x + B \cdot u$$

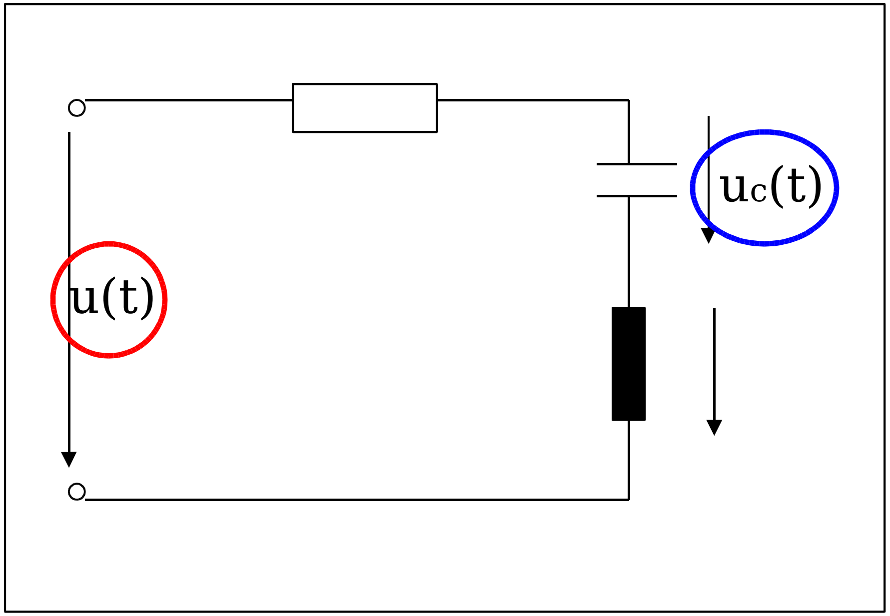
$$y = C \cdot x$$

Transferfunction

$$y(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots b_n s^n}{1 + a_1 s + a_2 s^2 + \ldots + a_m s^m} \cdot u(s)$$

real{zeros(poly)}<1

Example from before

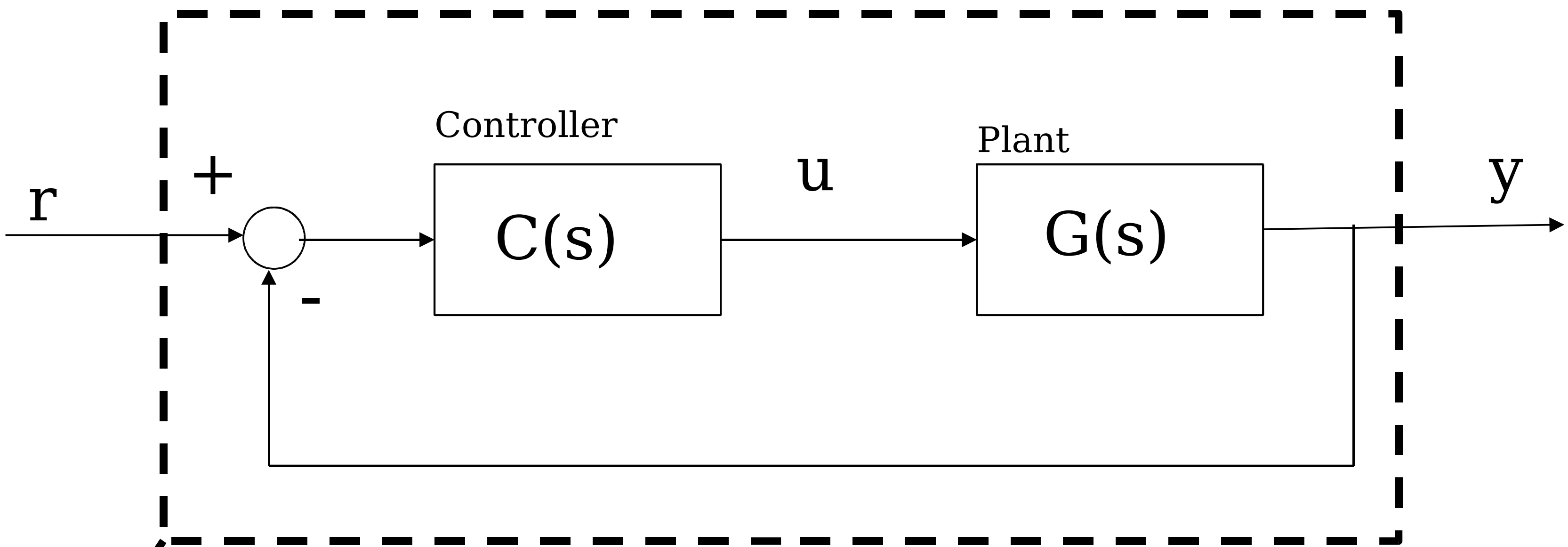


$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{bmatrix}-\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}\cdot\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}1\\ 0\end{pmatrix}\cdot u$$

$$y=\begin{pmatrix}0 & 1\end{pmatrix}\cdot\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$$\begin{bmatrix}-\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix} \longrightarrow \text{real\{eigvalues(A)\}<1}$$

The magic of feedback



$$T(s) = \frac{C(s) \cdot G(s)}{1 + C(s) \cdot G(s)}$$

$$C(s) = \frac{Q(s)}{R(s)}$$

$$G(s) = \frac{B(s)}{A(s)}$$

$$T(s) = \frac{Q(s) \cdot B(s)}{R(s) \cdot A(s) + Q(s) \cdot B(s)}$$

Knowledge needed to control

- mathematical model of the plant (open loop)
- define wanted properties (model) of the closed loop
- some extra constraints (depend on the problem)

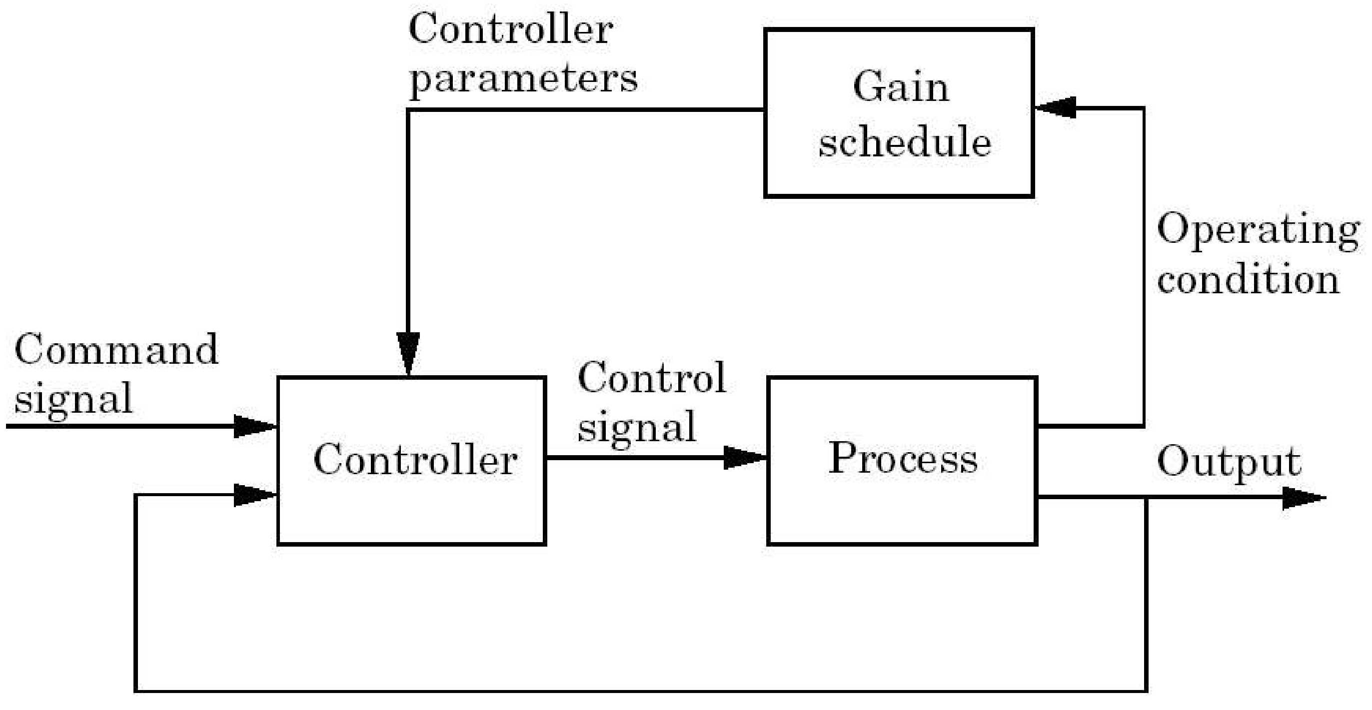
That's the big problem:
- complex models
- nonlinear models
- disturbances
- parameter drift, operation

One possible solution → **Adaptive Control**

Different Approaches

- Gain Scheduling
- Model Reference Adaptive Systems (MRAS)
- Self Regulating Tuners (SRT)
- Dual Control
- Nonlinear approaches

Gain Scheduling



Gain Scheduling

Used for:

- widely changing operation points (flight control)
- nonlinear actuators $v = f(u) = u^4$

→ Predictable variations

Key-Idea:

Finding rules between the changes of the model and significant variabels.

→ Feedback is nonlinear and maybe implemented as a look-up table.

Gain Scheduling

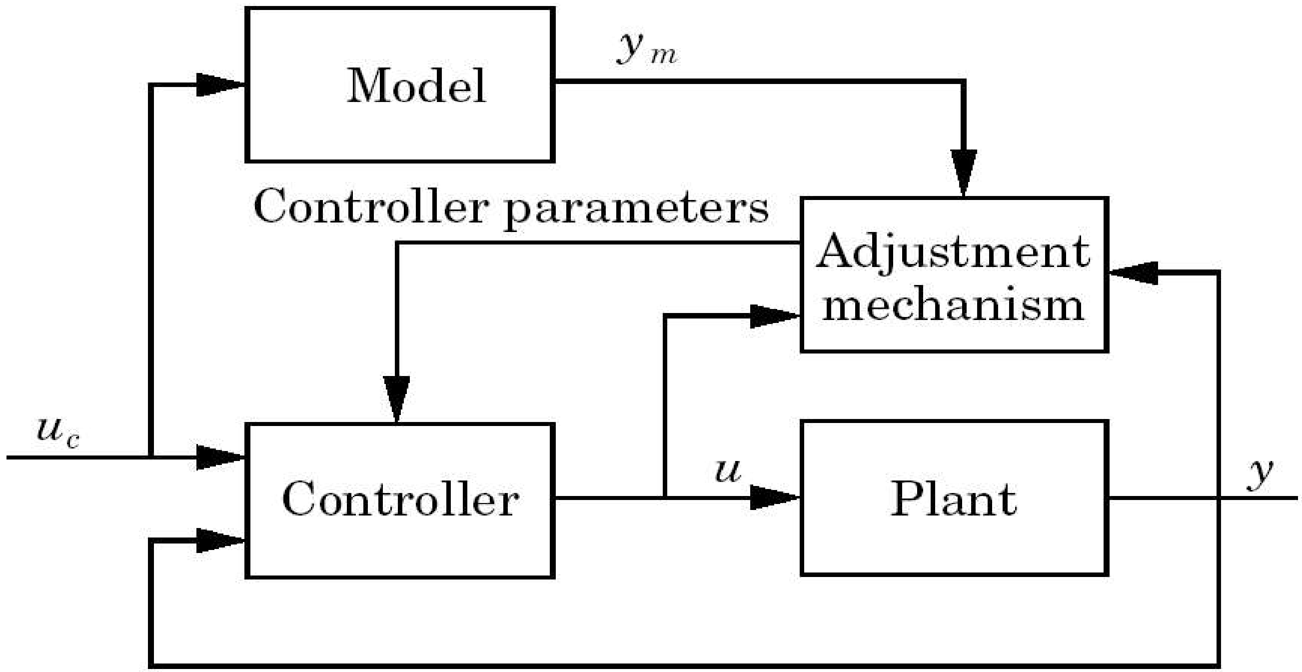
Advantages:

- very fast
- direct (no complex online calculations are needed)
- convenient if relation is known

Disadvantages:

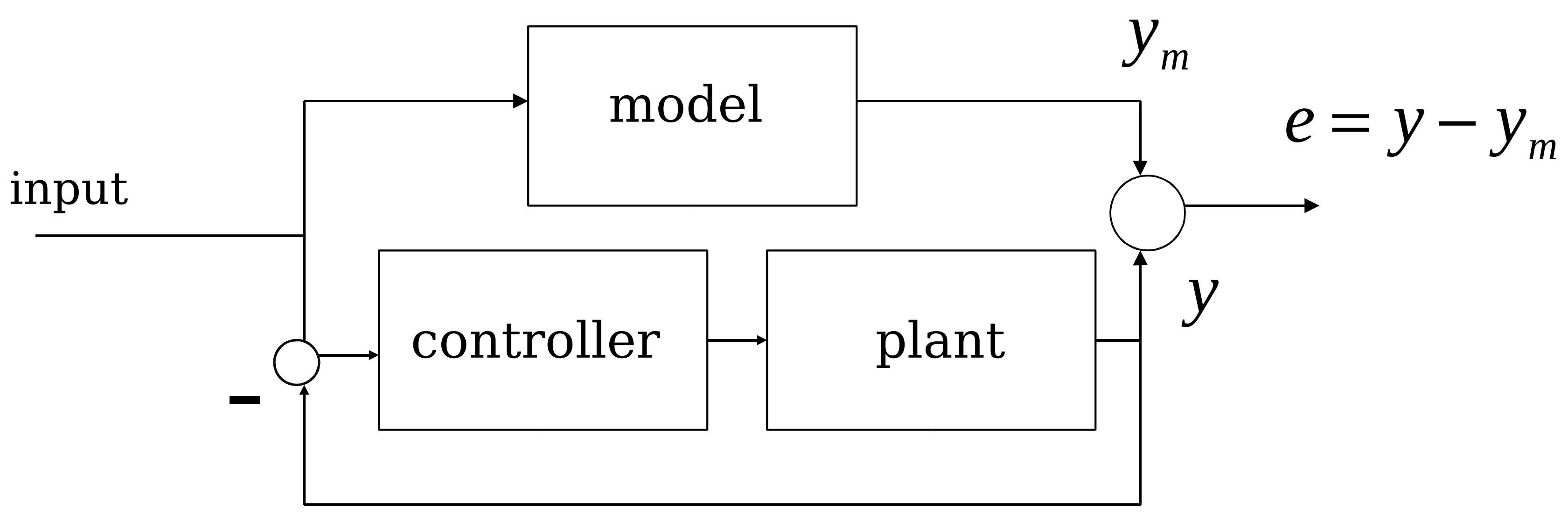
- a lot of knowledge about the plant is needed
- quite a lot of design work before starting
- not always applicable
- no real learning possibilities

Model Reference Adaptive Control



Model Reference Adaptive Control

key idea:	To express the wanted behaviour of the closed loop by defining a reference model.



MIT-Rule

Problem

Linear feedback of the error is not adequate for parameter adjustment.

Basic Idea:

Time derivation of the control parameters is proportional to the negative gradient.

$$e = y - y_m$$

→

$$J(\theta) = \frac{1}{2} e^2$$

→

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta}$$

MIT-Rule

$$e = y - y_m$$

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$$

$$\theta = f(G, G_m) = f(a, b, a_m, b_m)$$

Certainty equivalence principle !

$$G = G_m$$

$$a = a_m$$

$$b = b_m$$

MIT-Rule

Advantages

- easy principle
- convergence is proofed

Disadvantages:

- one has to know structure of the model
- stability problem
- choosing γ (small, but how small ?)

e.g. Theory of Lyapunov

Using Stability-Conditions

4 general steps:

1.) Determine a controller structure

2.) Derive the error equation

3.) Find a stability depending function

4.) Determine an adaptation law

MRAS - Summary

Advantages:

- easy formulation of behaviour due model

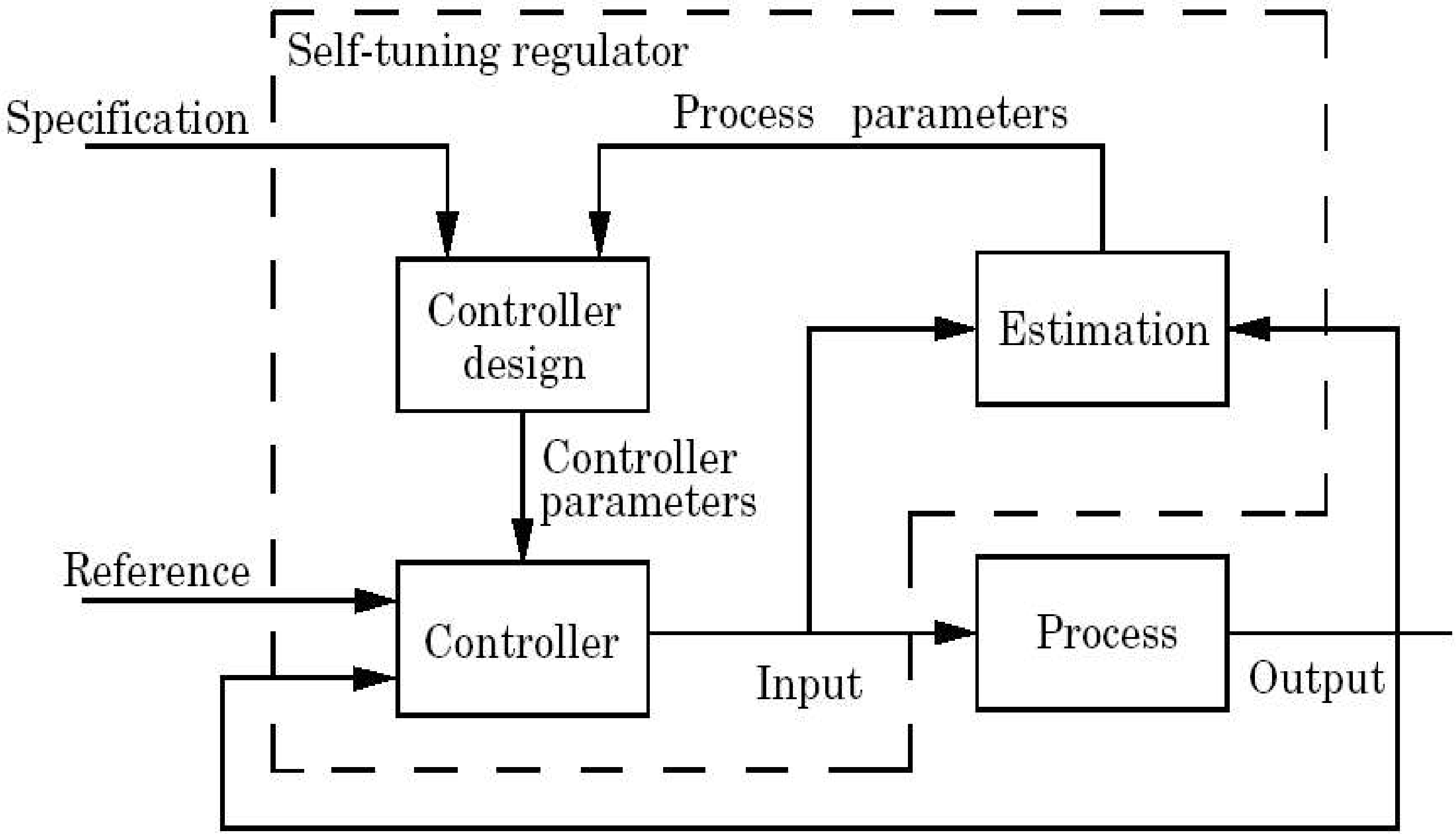
Disadvantages:

- some knowledge of the process needed
- finding adaptation law is non trivial

Algorithms

- MIT (normalized)
- Stability Design (BIBO, Lyapunov, Passivity)

Self Tuning Regulators STR



Self Tuning Regulators STR

Part 1:

Identification of the plant

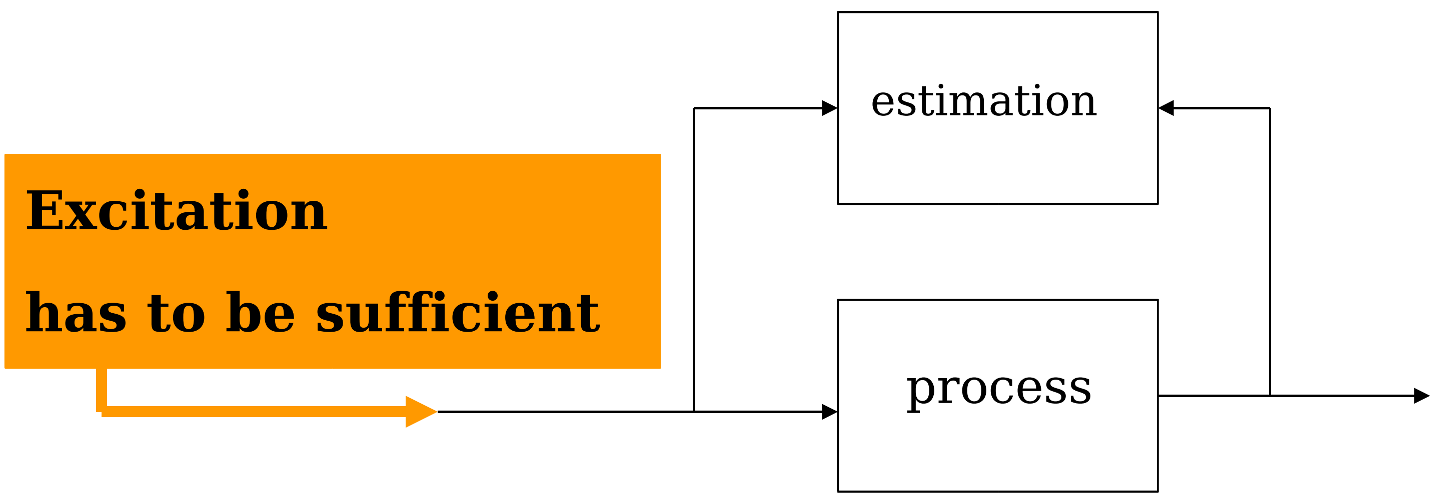
out of the input and output sequence

Certainty Equivalence Principle

Part 2

Using this model to design **online** a controller

STR - Part 1: Identification



Algorithms:

LMS (normalized)

- Projection algorithm
- RLS
- stochastic (ELS, Ext.Kalman)

Models:

linear (FIR, IIR, state space,...)

- ANN (feed-forward, recursive)
- Stochastic model
- and much more....

STR - Part 2: Controller Design

Possible design methods in principle all known methods:

But, has to :

- be fast enough (online calculation)
- able to check for stability
- take into account the nonlinear behaviour of adaptive systems
- converge together with identification process

Conclusions

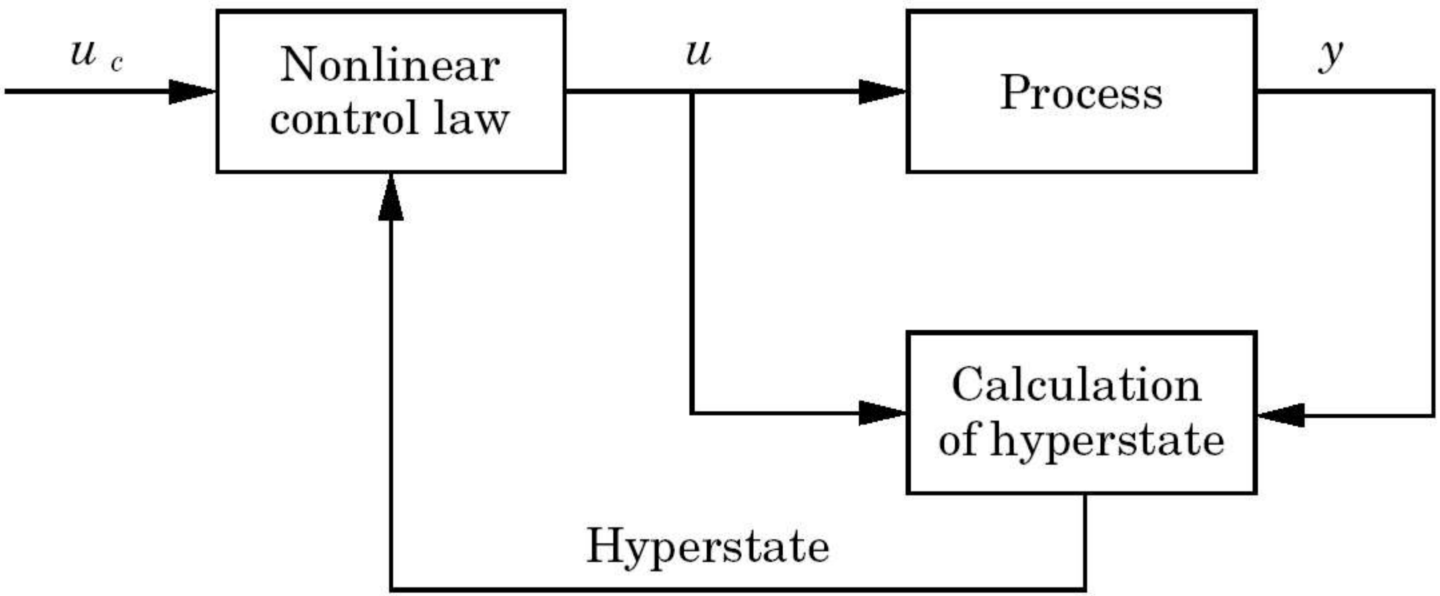
Advantages

- A lot of different possibilities for identification and for control design
- mathematical model isn´t necessary, but the general structure !
- easy to implement extra knowledge
- stochastic approaches are possible

Disadvantage

- complex system -> complex analysis
- excitation of input signal is necessary

Dual Control



Dual Control - a Stochastic Approach

no separation between state variables and model parameters

uncertainties of model parameters are taken into account

↓

leads to a hyperstate $z = \begin{pmatrix} x^T & \theta^T \end{pmatrix}^T$

↓

estimators generates a conditional probability distribution p(z|y,u)

↓

Minimization of a loss function

over control signal u(k)

$$V = \min_{u(k-1)\dots u(N-1)} E\big(f(z,u) \big)$$

↓

Dynamic Programming

Dual Control

- Solution is time consuming, if possible
- estimators generates a conditional probability distribution p(z|y,u)
- controller is a nonlinear mapping
    -> from hyperstate into controller parameters

Solution can have interesting properties:

- attempts to drive the output to the desired value (controller)
- introduce perturbations (probing) when parameter uncertain

Exploration - Exploitation

Dual Control

Advantages:

- elegant design
- conceptually interesting
- takes uncertainties of the estimation into account
- probing

Disadvantages:

- very complex
- solution very time intensive, if possible

Nonlinear Approaches - Overview

- Reparameterization
- Linearization
- Nonlinear feedback
- Nonlinear models (ANN, stochastic models)

Related field:

- Robust Control Design
- Predictive Control

Conclusion

- adaptive control are designed to control a varying processes
- adaptive systems are **nonlinea**r and **time variant** !!
- therefore analyse is quite complicated
- different approaches for different needs (no general theory yet)

further investigation needed:

- a general theory
- more stability proofs, convergence proofs
- get more insight into dual control (suboptimal solutions)

Thank you