Homework 5: VC-Dimension and Proofs

[Points: 12.5 , Issued: 2008/05/09 , Deadline: 2008/05/30 , Tutor: Gerhard Neumann ; Infohour: 2008/05/23 , 15:30-16:30 , HS i11 ; Einsichtnahme: 2008/06/13 , 15:30-16:30 , HS i11 ;]

5.1 Threshold circuits [8.5 points]

Prove that each Boolean function can be computed by a threshold circuit (Schwellenschaltkreis) of depth 2.

Remarks

- You can use the fact (without proving it) that there exists for each Boolean function $f : \{0,1\}^n \rightarrow \{0,1\}$ a Boolean formula in disjunctive normal form (DNF); see definition 1.1.5 of “Grundbegriffe der Aussagenlogik” for the precise definition of the DNF.

- Hence you just have to prove that there exists for each Boolean formula in DNF a circuit of threshold gates (of depth 2) which outputs 1 for each binary input vector if the DNF formula is true and outputs 0 if the DNF formula is false for the corresponding truth assignment.

5.2 VC-dimension [4 points]

- Consider the case of the hypothesis class of axis parallel rectangles. The VC-dimension of a single rectangle is 4. Find the best lower bound if you can use 2 rectangles instead of one !

- Consider the case of the hypothesis class of a single, axis parallel rectangle, where you can additionally choose whether to classify class 1 inside or outside the rectangle. What's the best lower bound for the VC-dimension you can proof ?