The impact of a synapse onto its postsynaptic neuron (amplitude of EPSPs/IPSPs) is termed the *weight* (efficacy) of a synapse.

This weight undergoes dynamics on a variety of time scales.

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Duration</th>
<th>Locus of Induction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short-term Enhancement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paired-pulse facilitation (PPF)</td>
<td>100 msec</td>
<td>Pre</td>
</tr>
<tr>
<td>Augmentation</td>
<td>10 sec</td>
<td>Pre</td>
</tr>
<tr>
<td>Post-tetanic potentiation</td>
<td>1 min</td>
<td>Pre</td>
</tr>
<tr>
<td><strong>Long-term Enhancement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term potentiation (STP)</td>
<td>15 min</td>
<td>Post</td>
</tr>
<tr>
<td>Long-term potentiation (LTP)</td>
<td>&gt; 30 min</td>
<td>Pre and post</td>
</tr>
<tr>
<td><strong>Depression</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paired-pulse depression (PPD)</td>
<td>100 msec</td>
<td>Pre</td>
</tr>
<tr>
<td>Depletion</td>
<td>10 sec</td>
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</tr>
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<td>&gt; 30 min</td>
<td>Pre and post</td>
</tr>
</tbody>
</table>

*Table 12.1. Different forms of synaptic plasticity*
All spikes have an approximately equal shape.

However, the PSPs of a synapse vary deterministically in the range of several 100 % in general.
The variation in $w(t)$ can be described as follows:

$$w(t) = w \cdot D(t) \cdot (1 + F(t))$$

$D(t) \in \{0, 1\}$, depression term

$F(t) \geq 0$, facilitation term
Examine not only rates of pre- and postsynaptic neurons, but precise spike times.

**Experiment:** Neurons A and B are forced to spike at times $t_{pre}$ and $t_{post}$ (pairing). The efficacy of the synapse changes as a function of $t_{pre} - t_{post}$. 
The synapse is \textit{strengthened}, if the presynaptic neuron fired in a certain time window \textit{before} the postsynaptic neuron.

The synapse is \textit{weakened}, if the presynaptic neuron fired in a certain time window \textit{after} the postsynaptic neuron.

The synapse gets information about $t_{\text{post}}$ through an action potential backpropagating the dendritic tree.
Spike train of neuron $i$ with $t_i^{(f)}$ as the $f$-th spike time:

$$S_i(t) = \sum_f \delta(t - t_i^{(f)}) .$$

The learning window $W$ defines the dependence of weight changes on the timing difference between the pre- and postsynaptic spikes:

$$W(s) = \begin{cases} 
A_+ e^{s/\tau_1} & \text{for } s < 0 \\
A_- e^{-s/\tau_2} & \text{for } s > 0 
\end{cases}$$
Model for learning dynamics:

\[
\frac{d}{dt} w_{ij}(t) = S^\text{pre}_j(t) \int_{0}^{\infty} W(s) S^\text{post}_i(t-s) \, ds + S^\text{post}_i(t) \int_{0}^{\infty} W(s) S^\text{pre}_j(t-s) \, ds
\]

Pre- and postsynaptic spike trains are drawn from a stochastic ensemble. Averages relatively to this ensemble are denoted by \( \langle \cdot \rangle_E \).

We compute the expected weight change over some (long) time \( T \):

\[
\frac{\langle w_{ij}(t + T) - w_{ij}(t) \rangle_E}{T} \approx \int_{-\infty}^{\infty} W(s) \langle \langle S^\text{post}_i(t-s) S^\text{pre}_j(t) \rangle_E \rangle_T \, ds
\]

\( \langle f(t) \rangle_T \equiv T^{-1} \int_{t}^{t+T} f(t') \, dt' \) is a temporal average.
Analysis of STDP

\[
\frac{\langle w_{ij}(t + T) - w_{ij}(t) \rangle_E}{T} \approx \int_{-\infty}^{\infty} W(s) \langle \nu_{ij}(t - s, t) \rangle_T ds
\]

Joint firing rate of neurons \(i\) and \(j\): \(\nu_{ij}(t, t') \equiv \langle S_{i}^{\text{post}}(t) S_{j}^{\text{pre}}(t') \rangle_E\).

- The mean weight change therefore depends on correlations between inputs and outputs.
- The output of the neuron is generated by the input.
- For simple neuron models, one can derive the weight changes given the statistics of the inputs (unsupervised learning).
- If the input is only weakly correlated with the output, the weight decreases.
- Groups of inputs which are strongly correlated (and drive the neuron) are strengthened.
- STDP tends to select inputs which are correlated on the timescale of the learning window.