Emergence of Complex Computational Structures From Chaotic Neural Networks Through Reward-Modulated Hebbian Learning

Gregor M. Hoerzer, Robert Legenstein and Wolfgang Maass

Institute for Theoretical Computer Science, Graz University of Technology, Graz, Austria

Address correspondence to Wolfgang Maass. Email: maass@igi.tugraz.at

This paper addresses the question how generic microcircuits of neurons in different parts of the cortex can attain and maintain different computational specializations. We show that if stochastic variations in the dynamics of local microcircuits are correlated with signals related to functional improvements of the brain (e.g. in the control of behavior), the computational operation of these microcircuits can become optimized for specific tasks such as the generation of specific periodic signals and task-dependent routing of information. Furthermore, we show that working memory can autonomously emerge through reward-modulated Hebbian learning, if needed for specific tasks. Altogether, our results suggest that reward-modulated synaptic plasticity can not only optimize the network parameters for specific computational tasks, but also initiate a functional rewiring that re-programs microcircuits, thereby generating diverse computational functions in different generic cortical microcircuits. On a more general level, this work provides a new perspective for a standard model for computations in generic cortical microcircuits (liquid computing model). It shows that the arguably most problematic assumption of this model, the postulate of a teacher that trains neural readouts through supervised learning, can be eliminated. We show that generic networks of neurons can learn numerous biologically relevant computations through trial and error.

Keywords: cortical microcircuit model, cortical plasticity, pattern generation, working memory

Introduction

Generic networks of neurons in different locations of the cortex perform a large variety of different computations and pattern generation tasks. It has remained an open problem how these diverse computational functions are attained and maintained, in spite of the generic laminar architecture of cortical microcircuits in different cortical areas. A large body of experimental work suggests that cortical function is adapted during learning in order to optimize performance. Populations of neurons in the lateral prefrontal cortex change their response properties during a delayed matching-to-sample task that involved noise-degraded visual stimuli in a way that correlates with performance improvements (Rainer and Miller 2000). Repeated training of working memory tasks can improve performance (Klingberg et al. 2002) and this improvement can transfer to tasks that were not a part of the training program (Klingberg 2010). Functional magnetic resonance imaging studies showed that such training is accompanied by increased activity in the prefrontal and parietal cortex (Olesen et al. 2003). It has already been shown that networks of spiking neurons with stereotypical connection probabilities can in principle support a large variety of computational tasks (Maass et al. 2002; Haeusler and Maass 2007; Haeusler et al. 2008; Buonomano and Maass 2009). In these models, termed liquid computing models, the synaptic weights of readout neurons that project to other circuits or areas are modulated by synaptic plasticity. The aim of synaptic adaptations is to approximate a desired output signal with the actual output of these readout neurons. Since these synaptic plasticity rules require knowledge of the desired output signal, this learning process is referred to as supervised learning, where a postulated supervisor or teacher supports the learning process. Furthermore, it has been shown that if such supervised learning is applied to readout neurons whose axons are also connected to other neurons within the network, such generic networks can also learn to generate periodic patterns and working memory (Jaeger and Haas 2004; Maass et al. 2007; Sussillo and Abbott 2009). These results imply that, in order to “program” the network to carry out a specific type of computation, it suffices to assign suitable values to the synaptic weights of some neurons. Furthermore, local synaptic learning rules are able to produce suitable weight settings, provided that the desired output (target output) of each readout neuron is provided at any moment during training by a teacher or supervisor. But for most concrete tasks, the assumption of such a teacher signal requires that there already exist some other neuron or network that is able to perform the desired computational task. Hence, this set-up is more suitable for duplicating a computational function, rather than for explaining how it could emerge in the first place.

We begin in this paper a new chapter in liquid computing theory, by investigating what computational properties can emerge in this approach if one eliminates all supervised learning; that is, all learning rules that require a teacher that tells a neuron how it should respond at any given time. We show that, for a large class of biologically relevant computational tasks, a teacher signal is not needed. It can be replaced by a biologically more realistic signal that assumes a high value if the average performance has recently increased and a low value otherwise. In fact, we show that several different tasks can be learned by different readouts simultaneously, based on such a relatively uninformative global feedback signal about average performance improvements. This set-up requires each neuron to explore different output values for the same
network inputs. Hence, our model is not only consistent with experimentally observed stochastic features of neuronal responses (trial-to-trial variability), but requires these stochastic features for its function. In this article, for the sake of simplicity, we are discussing only rate models for generic microcircuits. Mean field models predict that sufficiently large populations of spiking neurons will behave very similar to these rate-based models (see Discussion).

The learning rule that we apply is a variation of the exploratory Hebbian (EH) rule from Legenstein et al. (2010). The EH rule is a 3-factor learning rule (Kandel and Tauc 1965a, 1965b; Bailey et al. 2000; Fiete and Seung 2006; Fremaux et al. 2010; Pawlak et al. 2010) that depends on—besides pre- and postsynaptic neural activity—a modulatory third factor. A large set of experimental data shows that neuromodulators, such as dopamine, implement such a third factor in biological networks of neurons (Reynolds et al. 2001; Reynolds and Wickens 2002; Pawlak et al. 2010), but also a modulatory signal in the form of synaptic input can influence the amplitude of the backpropagating action potential of a neuron, and thereby the learning rate of spike-timing dependent plasticity (Waters and Helmbchen 2004; Sjöström and Häusser 2006). In this paper, we employ a variation of the EH rule that imposes extremely weak demands on the information provided by this third factor. Rather than assuming that it provides information regarding how much—and in which direction—the current system response deviates from some hypothetical target response, we only assume here a 2-valued global third factor \(M(t)\). This global signal informs all local plasticity mechanisms whether the system performance has recently improved. We investigate in this paper to what extent this—arguably the least informative performance-related third factor that one can conceive—suffices for installing in generic recurrent networks of neurons different, task-dependent, computational organizations. 

We consider 4 different types of computational tasks:

1. Periodic pattern generation.
2. Learning a rule, which requires storage of specific information in working memory, and application of this stored information for online computation on complex analog input signals.
4. Nonlinear analog computations on complex input signals.

In spite of the heterogeneity of these 4 computational tasks, we show that they can all be learned by the same generic neural circuit, requiring only the previously described weak information about recent performance improvement. There exists substantial evidence that neural networks of primates and other animals can carry out these tasks (see Discussion). But it has remained an open problem how biological neural networks could acquire these specific computational capabilities. 

In summary, the results of this work provide a new model for the emergence of diverse complex computations in biological neural systems.

The remainder of this paper is structured as follows. We first introduce the generic microcircuit model and the reward-modulated plasticity rule used in section Materials and Methods. In Results, we then show how this network can autonomously learn to generate a complex periodic pattern. The network performance is compared with the performance of previously studied models, and the robustness of network function to various types of noise perturbations is demonstrated. We then test the same network on a completely different task that requires the emergence of 2 independent working memory stores. In a final simulation task, we show that the same network can acquire the ability to dynamically route information in a state-dependent manner. Several predictions of our model are discussed in Discussion.

Materials and Methods

Network Architecture

We employed a generic network model consisting of \(N\) sparsely recurrently connected neurons (with a connection probability of 0.1). We refer to these neurons as the network neurons in the following. The recurrent network model is generic in the sense that it is not designed for a particular computational function. Instead, connections within the network are randomly drawn such that network neurons are sparsely connected by excitatory and inhibitory synapses (cf. Supplementary Methods). Similar network models have been previously used to model the dynamics of recurrent biological networks of neurons (Amari 1972; Hopfield 1984; Haykin 1999; Sussillo and Abbott 2009). If necessary for the computational task, some network neurons receive in addition projections from external input streams \(u_j(t)\). Specific computational functions are acquired through synaptic modification of the weights from neurons in the network to so-called readout neurons, which could represent, for example, layer 5 pyramidal cells (Maass et al. 2002; Jaeger 2003). These readout neurons can also feed back their activity into the recurrent neural network (Jaeger and Haas 2004; Maass et al. 2007). Figures 1A and 3A show the basic network topology without and with external input streams, respectively.

In our model, the state \(x_j(t)\) of neuron \(j\) represents its membrane potential at the soma at time \(t\), resulting from excitatory and inhibitory synaptic inputs (Hopfield 1984; Haykin 1999). The firing rate of the \(j\)th neuron at time \(t\) is given by

\[
r_j(t) = \tan h[x_j(t)] + 6^{\text{state}}(t),
\]

where \(6^{\text{state}}(t)\) models zero-mean noise on the firing rate of the neuron (see Supplementary Methods for details on noise statistics). The network dynamics is given by

\[
r_j(t) = \lambda \sum_{i=1}^N W_{ij}^{\text{rec}} r_i(t) + \sum_{i=1}^M W_{ij}^{\text{in}} y_i(t) + \lambda \sum_{i=1}^L W_{ij}^{\text{fb}} z_i(t),
\]

where \(\lambda\) is the membrane time constant. The parameters \(W_{ij}^{\text{rec}}, W_{ij}^{\text{in}},\) and \(W_{ij}^{\text{fb}}\) denote the synaptic weights for recurrent connections within the network, connections from inputs to the network, and feedback connections from readout neurons to the network neurons, respectively.

Different dynamic regimes, from ordered to chaotic, can be accomplished by scaling the recurrent synaptic connections through the parameter \(\lambda\). We choose a value of \(\lambda\) so that, as in Sussillo and Abbott (2009), the dynamics of the recurrent network tend to be in the chaotic regime prior to learning. During learning, the readout neurons drive the network activities into a nonchaotic regime via the feedback pathway. Supplementary Figure 1 illustrates the contributions of input connections, recurrent connections, and feedback connections to the membrane potential of network neurons.

We assemble the firing activities (rates) of the network neurons at time \(t\) into a column vector \(\mathbf{r}(t)\). Assembling the synaptic weights of connections from these neurons to a readout neuron \(i\) in a corresponding column vector \(\mathbf{w}_i\), this readout neuron \(i\) computes the function

\[
\hat{z}_i(t) = \mathbf{w}_i^T \mathbf{r}(t) + b_i.
\]
Here, the bias $b_i$ models the baseline activity (spontaneous activity) of the neuron. Our tasks do not necessitate the bias term, but we chose to adopt it for consistency with Legenstein et al. (2010). Reward-modulated Hebbian learning of readout weights requires the readout neurons to be noisy (Legenstein et al. 2010). Therefore, the output of the $i$th readout neuron is modeled by

$$z_i(t) = \hat{z}_i(t) + \xi_i(t),$$

where $\xi_i(t)$ models zero-mean exploration noise on the firing rate of
readout neuron $i$. We did not apply the tanh function for readout neurons to match the model of Sussillo and Abbott (2009). In some cases, this noisy readout signal is also fed back to the network neurons. One can apply exploration noise $\xi(t)$ either only during learning (then $z_i(t) = \tilde{z}_i(t)$ during a subsequent testing period), or during learning and testing, yielding a similar performance.

### Reward-Modulated Learning Rule

In contrast to Sussillo and Abbott (2009), who use a fully supervised online learning rule to train the network, we investigate the capabilities of a biologically more plausible reward-modulated online learning rule. The supervisor is replaced by a binary signal $M(t)$ that communicates whether the average performance of all readouts (which are generally required to carry out different computational tasks) has recently increased. But it does not provide information on the sign and magnitude of the error, nor on which readouts contributed how much to a recent change in the performance. This single modulatory signal, which could be viewed, for example, as an abstract model for the phasic output of dopaminergic systems in the brain (Schultz et al. 1997; Schultz 2007), modulates synaptic plasticity of all synapses of all readout neurons in our simple model.

More precisely, the modulatory signal $M(t)$ is defined by

$$M(t) = \begin{cases} 1 & \text{if } P(t) > \bar{P}(t) \\ 0 & \text{if } P(t) \leq \bar{P}(t) \end{cases}$$

(5)

where $P(t)$ is the current performance of the system and $\bar{P}(t)$ is a low-pass filtered version of $P(t)$ that reflects its recent performance (see Supplementary Methods for the implementation of the low-pass filter).

The current system performance $P(t)$ is given by the sum of the mean squared errors (MSEs) of all readout neurons:

$$P(t) = \frac{1}{N} \sum_{i=1}^{N} \left( z_i(t) - f_i(t) \right)^2,$$

(6)

where $f_i(t)$ is the target output of readout neuron $i$ for its assigned computational task. It is important to note that the values $f_i(t)$ of the target signals are not communicated to the neural network. This is appropriate, since their values may be unknown to a biological organism, while the existence of solutions $f_i(t)$ is assured through evolution.

We employ a variant of the EH rule proposed by Legenstein et al. (2010), where the weight change $\Delta w_i(t)$ of readout neuron $i$ at time $t$ is given by

$$\Delta w_i(t) = \eta(t) [z_i(t) - \tilde{z}_i(t)] M(t) x_i(t).$$

(7)

Here, $\eta(t)$ is a small learning rate that can either be constant, or decay over time, such that learning saturates as learning progresses. We use a decaying learning rate in all simulations reported in this paper. See Supplementary Results for additional simulation results with constant learning rates and for simulations used to determine the choice of the learning rate. Here $z_i(t)$ is a low-pass filtered version of the noisy output $\tilde{z}_i(t)$ of the readout (cf. Supplementary Methods). If one assumes that $\tilde{z}_i(t)$ changes only slightly within the time scale of the filter, and the noise is only weakly correlated over time, then the term $z_i(t) - \tilde{z}_i(t)$ approximates the noise $\xi_i(t)$. Therefore, in contrast to previously proposed rules (Fiete and Seung 2006), the rule does not need explicit information on the exploration noise, but instead estimates the noise autonomously (Legenstein et al. 2010).

This learning rule is Hebbian, since it uses the correlation of changes in the postsynaptic activity and the activity of the presynaptic neuron. It belongs to the category of reward-modulated Hebbian learning rules (Bailey et al. 2000; Legenstein et al. 2008; Fremaux et al. 2010; Pawlak et al. 2010), because this correlation is multiplied with the modulatory signal $M(t)$. Since our goal was to demonstrate learning in a minimal model, we used in our simulations a variant of the original EH rule, where the modulatory signal assumes only 2 possible values. From a biological perspective, this synaptic plasticity rule only distinguishes between a high and a low modulatory signal (that could, for example, be a high or low concentration of some neuromodulator). When compared with the original formulation in Legenstein et al. (2010), where $M(t)$ was chosen to be $P(t) - \bar{P}(t)$, this signal is much less informative. It does not communicate the magnitude of the performance change from the recent past but only whether it has improved at all. Additional simulations with the analog modulatory signal $M(t) = P(t) - \bar{P}(t)$ indicate that the performance of the 2-valued signal is comparable with the analog one; see Supplementary Results.

We point out that the exploration noise $\xi_i(t)$ is the driving force of learning. Without perturbations of the readouts’ output, no learning would take place.

In all our simulations, we used $N=1000$ neurons in the recurrent network. These and other basic network parameters were chosen such that they correspond to the values used in Sussillo and Abbott (2009). We fixed suitable values for the other parameters that we kept for all simulations. In other words, we did not perform a parameter search in order to find the set-up with optimal performance for each individual simulation task (see Supplementary Methods for details on the parameter setting).

### Results

#### Autonomous Learning of Periodic Pattern Generation

Biological neural networks produce many different types of rhythmic activities for various purposes, such as muscle activations, breathing, or locomotion. Sussillo and Abbott (2009) showed that a desired rhythmic activity can be acquired by generic recurrent neural circuits through supervised learning, where the desired output of each readout is provided at any moment during learning by a teacher or supervisor. The existence of such a teacher signal implies that some other neuron or network exists that is able to perform the task. Hence, this set-up cannot explain how the computational function could emerge in the first place. We therefore studied whether such tasks can also be learned autonomously without a teacher. We replaced the teacher signal by a modulatory signal $M(t)$ that indicates whether the performance of the neural circuit for the considered task recently increased; see Equation (5) in Materials and Methods. The supervised learning rule used in Sussillo and Abbott (2009) was replaced by reward-modulated Hebbian learning, that is, by Hebbian synaptic plasticity that is modulated by the modulatory signal $M(t)$; see Equation (7) in Materials and Methods. We simulated a network that receives no inputs besides the feedback projections from a single readout neuron (Fig. 1A). The task of the readout neuron was to produce a specific periodic trajectory and to repeat this periodic trajectory in a stable manner.

Since the actual output of the readout and not the target signal is fed back into the network during learning, the readout output has to resemble the target computational function already shortly after the beginning of learning (Sussillo and Abbott 2009). Figure 1B shows a representative example of the readout activity at the onset of the learning procedure. Within less than 50 ms, the readout is able to adapt its activity in order to reach the desired target, and to approximately follow the target function henceforth. The goal of learning is to find a set of time-independent weights such that the system is able to keep producing the target function when the learning mechanism is switched off after an appropriate learning time. Figure 1C shows that this goal is accomplished by reward-modulated Hebbian learning on the synapses from the
network to the readout neuron. After a learning time of 400 s, which corresponds to 400 oscillation cycles of the periodic trajectory, the readout keeps producing the desired trajectory in spite of the lack of any further weight adaptation. Here, no exploration noise was applied during testing. However, the performance is similar when exploration noise is applied during testing as well (cf. Supplementary Fig. 2A). After learning, one can usually observe a small drift from the desired trajectory over time, as depicted in Figure 1D. This drift arises due to the fact that the oscillation length of the frequency components of the readout output is not perfectly matched to the frequency components of the desired target signal. Such drift is to be expected irrespective of the applied learning mechanism. However, the difference in the oscillation length is very small (Fig. 1H). If an animal reproduces a periodic pattern, for example, for locomotion, performance depends on how well the shape of the pattern is reproduced, but the cumulative effects of the drift can generally be ignored. We therefore corrected for the drift in the subsequent performance evaluations. This was done by cutting the readout’s output during the testing interval into successive time slices of 1 s and by calculating the minimum MSE between each time slice and circularly shifted versions of a 1-oscillation cycle slice of the target pattern (Supplementary Methods).

The rhythmic activity of the readout, which drives the network via the feedback pathway, has a strong influence on the internal network dynamics. Figure 1E,F shows a subset of 5 random units within the network at the onset of learning and at the transition from learning to testing, respectively. Before the system starts with the learning process, the network exhibits chaotic dynamics and produces rich spontaneous activity. Shortly after the onset of learning, a stable periodic pattern emerges due to the driving force of the feedback loop (panel E). This stable periodic pattern persists during the testing interval when there is no further weight adaptation (panel F). A certain level of chaoticity—which we regulate by the parameter $\lambda$ that scales the weights of the recurrent network—is necessary for an accurate performance of the system. Initial chaotic dynamics are needed because the network has to initially produce sufficiently rich dynamics to properly generate the target function. On the other hand, if the chaoticity exceeds a certain level, the drive from the feedback loop is too weak to drive the network dynamics into a stable regime (Supplementary Fig. 2D; see also Supplementary Fig. 1 for the contributions of input connections, recurrent connections, and feedback connections to the membrane potential of network neurons). This is consistent with the results of Sussillo and Abbott (2009).

To investigate the learning time needed such that the network reliably autonomously reproduces the oscillatory pattern, we conducted 50 independent simulation trials with different learning times from 10 to 400 s. Each simulation trial consisted of a learning interval of varying length and a subsequent testing interval of 500 s. Moreover, in order to evaluate whether the elimination of the teacher leads to a significant decrease in performance, we conducted the same simulations with systems employing 2 different FORCE learning rules and compared the performance of the systems (cf. Supplementary Methods for a brief description of the FORCE learning rules). The supervised FORCE learning rules have previously been tested for readout training on similar tasks (Sussillo and Abbott 2009). Figure 1G shows the result of this comparison. The recursive least squares (RLS)-based FORCE rule (green) performs best, leading to a good approximation of the target signal after learning for as few as 10 s (which corresponds to 10 oscillation cycles of the target pattern). This is not surprising since the RLS-based rule uses nonlocal information about the correlations between all pairs of inputs to the readout to adapt the individual synapses. However, this approach seems to be problematic from the point of biological plausibility. The reward-modulated Hebbian learning rule (red) performs similarly to the local least mean squares (LMS)-based FORCE rule (blue) that still requires full knowledge of the desired output signal. With both of these local learning rules, the network needs to learn for approximately 100 s before a good performance level is reached. The desired trajectory was stably produced until the end of the testing interval in the majority of simulation trials (cf. Supplementary Fig. 2C). A comparison of the performance distributions across simulation trials showed that the performance of our learning rule did not differ significantly from the performance of the LMS FORCE rule for learning times of $>100$ s. The performance of our learning rule remained approximately constant across learning times $>150$ s (i.e., there was no significant difference between the performances at any 2 training times $>150$ s; nonparametric Wilcoxon rank-sum tests, significance level: 5%). For the comparison between the LMS FORCE rule and the EH rule, learning rates were independently set to good values that were obtained by a brute force search (see Supplementary Results for the choice of the learning rates for the 2 rules).

Biological networks of neurons have to be able to operate under the influence of substantial amounts of noise and other perturbances. Therefore, we have tested the noise robustness of the model. The results show that the system, which has learned without a supervisor, is indeed robust to noisy perturbations of the network state and the readout output as well as to long clamping of the readout output after learning. Additionally, substantial amounts of noise can also be applied to network neurons during learning (Supplementary Results).

**Necessary Conditions**

To investigate the limits of this approach, we tested the system behavior under various conditions. We performed simulations where we varied some of the system parameters and the properties of the target pattern. Three factors were varied concurrently, leading to a total of 45 parameter settings for which the network was tested (Fig. 2). Specifically, we investigated the influence of the following network and target pattern properties on the ability of the network to generate a periodic target pattern:

1. Frequency components of the target pattern (columns in Fig. 2A);
2. Update interval of the weights and the modulatory signal $M(t)$ (rows in Fig. 2A); and
3. Time constant of the exploration noise (x-axis of histograms in Fig. 2A).

The frequency components and therefore the rate of change of the target pattern are an important factor for the difficulty of the task. If the target signal changes too fast, the readout is not able to adapt its output quickly enough to follow the
target pattern during learning. Additionally, in order to assure that temporal averages are estimated with sufficient accuracy in the EH rule, the noise-free readout activation should change slowly on the time scale of these averages (see Legenstein et al. 2010 for details). Each column of Figure 2A corresponds to simulations with a set of given frequency components. The periodic target pattern in the above simulations had a length of 1 s with frequency components between 1 and 4 Hz (middle column). We performed simulations where this target pattern was scaled to lengths of 0.5 and 2 s.
corresponding to frequencies of 2–8 Hz (right column) and 0.5–2 Hz (left column), respectively. As expected, learning works well if frequency components are sufficiently slow (up to 4 Hz in our simulations). Performance for patterns with higher frequency components can be strongly improved if the network time constant \( \tau \) is reduced (cf. Supplementary Fig. 3).

It was assumed in the above simulations that the modulatory signal \( M(t) \) is provided at every time step. Based on this signal, the synaptic weights to the readout neurons were updated as frequently. The frequency of the weight update and update of \( M(t) \) is a crucial parameter because weight updates have to be sufficiently fast compared with the system evolution in order to be able to adapt to the target trajectory quickly after the onset of learning and to follow the target trajectory during the learning process. If the system is not driven into the expected regime by the feedback signal, the corresponding transformation from the network state to the desired output trajectory cannot be learned properly by the readout. Each row in Figure 2A corresponds to simulations with a given update interval: 1 ms (top row; as in our initial simulations), 5 ms (middle row), and 10 ms (bottom row). The results indicate that longer update intervals are still possible if frequency components in the target pattern are sufficiently slow. As expected, learning of fast frequency components is not possible if update intervals are long.

In the theoretical justification of the EH rule, it is assumed that the exploration noise in the readout neurons is not (or at least only weakly) correlated over time (Legenstein et al. 2010). This may not always be justified in biological readout neurons. Temporally correlated exploration noise is expected to lead to worse performance since the readout neuron can explore fewer settings in a given time interval, which again leads to slower adaptation of the readout output. We tested the stability of the system against temporal correlations in the exploration noise by performing simulations with exploration noise that is temporally correlated with time constants \( \tau_{\text{noise}} \) between 1 and 5 ms (Supplementary Methods). In Figure 2A, the \( x \)-axis of each of the 2-dimensional histograms is dedicated to the exploration noise time constant. As expected, low noise correlations are beneficial for the learning process. With increasing frequencies in the target pattern, the system is likely to fail to reach appropriate performance levels if the exploration noise is temporally correlated (upper right panel). For additional simulation results with different target patterns, see Supplementary Results.

**Autonomous Learning of a Computational Rule and of the Working Memory that it Requires**

Many cognitive operations of the brain require a working memory, where specific task-relevant information is stored for intervals up to a few seconds. Neuronal correlates of working memory have been observed, for example, in single neuron recordings from the prefrontal cortex of macaque monkeys during visual working memory in delayed matching-to-sample tasks (Fuster and Alexander 1971; Goldman-Rakic 1995; Miller et al. 1996; Bernacchia et al. 2011). In these experiments, it was observed that prefrontal cortex neurons hold information of previously observed stimuli by a persistent increase or decrease of their firing rates for a time interval in the range of seconds.

We tested whether such memory-dependent processing can emerge in our model through reward-modulated learning. We designed a task where good performance could only be achieved when the network state retained specific information about the input history. The task required the output value \( z_1(t) \) of the first readout neuron to be high [target value \( f_1(t) \)] when—among the 2 input streams \( u_{\text{on},1}(t) \) and \( u_{\text{off},1}(t) \)—the most recent high activity had occurred in stream \( u_{\text{on},1}(t) \); otherwise the readout value \( z_1(t) \) was required to be low. The complexity of the task was increased substantially by adding a second readout neuron \( z_2(t) \) that was expected to learn independently the corresponding task for 2 further input streams \( u_{\text{on},2}(t) \) and \( u_{\text{off},2}(t) \). The performance \( P(t) \) of the whole system was given by the sum of the MSEs of the 2 readouts from their (implicit) target values; see Equation (6). As in the preceding simulation task, the network only received information whether this performance had recently improved through the modulatory signal \( M(t) \). Thus, it neither received information about the target value of any of the readouts, nor
in case of a performance improvement signaled by $M(t) = 1$—about which of the 2 readouts’ performance had improved. This global signal $M(t)$ was provided as the third factor to the synaptic learning rules of all synapses of the 2 readout neurons. To reach good performance, the network had to learn from this global signal $M(t)$ both the computational rule for the task (simultaneously for both of the 2 readout neurons), as well as to establish 2 independent nonfading working memories, one for the task-relevant information from the history of inputs $u_{on, i}(t)$ and $u_{off, i}(t)$, and another for the inputs $u_{on, 2}(t)$ and $u_{off, 2}(t)$.

Figure 3A shows the set-up for this task. Figure 3B shows a representative example of network performance after 500 s of learning. The last 30 s of a subsequent 500-s testing interval are presented. The firing rates of the 2 readouts correctly change depending on the associated inputs. Hence, the system learned the correct behavior for both readouts based on a single modulatory signal, which indicated only whether the combined performance of both readouts recently improved. We performed 50 independent simulation trials and calculated the percentage of time in which the readouts are in the wrong state (output value closer to 0.5 rather than −0.5, or vice versa) during a 500-s testing interval. The average fraction of time in the wrong state is $4.57 \pm 0.73\%$ of the testing time, with an even lower median fraction of $2.9\%$ (based on 50 simulation trials, average proportion over both readouts per simulation trial).

To analyze the behavior of the network in this task, we performed a principal component analysis (PCA) of the network activity vectors $r(t)$ before learning and during the test epoch. Figure 4 shows the network trajectories in this task projected onto the first 3 principal components of the test trajectories before learning (panel A), early during learning (panel B), and during testing (panel C). As expected, before learning starts, the network dynamics are not restricted to specific attractor subregions (panel A) and network dynamics are high dimensional (panel D). Early in the learning process, the feedback from the readout neurons—which is constantly adapted by the EH learning rule—already constrains the network to a low-dimensional subspace (panel D). We identified 4 subareas that are visited with likelihoods that depend on the momentary readout state (panel B). This behavior is preserved after learning (panel C, D). This indicates that the adaptation of the readout weights autonomously generates attractor regions in the activity landscape of the network to implement the working memory necessary for this task. Pulses at the network inputs move the activity state of the network to the corresponding region.

In summary, these simulations show that a generic neural circuit can learn a computational rule and simultaneously hold information about recently observed inputs for at least several seconds. This learning took place without any supervision, just from information about recent changes in global system performance.

**Emergence of Context-Dependent Switchable Routing of Information**

It has frequently been conjectured that networks of neurons in the brain are able to route information in a task- and context-dependent manner between relevant brain areas, giving rise to “effective connectivity” as opposed to “structural” network connectivity. But it has remained an open problem how this can be achieved (or even be learnt). We show in our last task that the same generic neural circuit, with the same general purpose reward-modulated Hebbian learning rule as in our preceding simulation tasks, can achieve this.
The network (Fig. 5A) receives here 4 input streams \( u_3(t), \ldots, u_4(t) \). The first 2 play the same role for the computational task of the first readout \( z_4 \) as before: They represent a switchable state (context), henceforth denoted by state “on” or “off.” The first readout has to learn to maintain a working memory of this context. The other 2 inputs \( u_3(t) \) and \( u_4(t) \) are 2 independently generated generic time-varying analog signals (Supplementary Methods). We require that the network learns to route the signal \( u_3(t) \) to the second readout \( z_2 \) if the network is in state “on”, and to route the signal \( u_4(t) \) to this second readout \( z_2 \) if the network is in state “off.” To test whether the network can simultaneously learn to carry out a demanding state-independent computation on the same 2 input signals \( u_3(t) \) and \( u_4(t) \), the global performance measure \( P(t) \) also included a third readout \( z_3 \) that was required to learn a complex nonlinear online computation on these 2 input signals, and to output \( f_3(t) = 0.5[u_3(t)^2 + u_4(t)^2 + u_3(t) u_4(t)] \). While the routing task could have been performed similarly without this third computation, we introduced it in order to make the task even more difficult and to show that, despite the feedback from the memory providing readout, the network state was still high-dimensional enough to be able to carry out such computation. Altogether, the performance measure \( P(t) \) was defined as the sum of the MSEs of the 3 readouts from their (implicit) target values; see Equation (6). As before, the network (or more precisely: the learning rules for the synapses of the 3 readouts) received through the global signal \( M(t) \) only the information on whether this composite performance function \( P(t) \) had recently improved.

Figure 5B,C shows the readout outputs of the network at the end of a 500-s testing interval, after 500 s of learning. The first trace in Figure 5B shows an example in which the transition of the memory unit from the “on” state to the “off” state is correctly executed. At the time of this state transition, the routing unit also changes its output from approximately representing the input \( u_3(t) \) (blue) to representing the input \( u_4(t) \) (green). The third readout is not affected by the state switch and correctly computes the nonlinear function of both of these inputs throughout. This shows that a generic neural circuit is able to learn to perform concurrently complex memory-dependent operations and memory-independent nonlinear computations. Figure 5C shows an example from the same trial, but approximately 50 s earlier, where a switch of the first readout to the “on” state failed. The second readout behaves as expected: Since the first readout is in the wrong state, the second readout also represents the “wrong” input \( u_4(t) \) (green) instead of \( u_3(t) \) (blue). As soon as the first readout accurately switches to the “on” state, the second readout also switches to representing \( u_3(t) \) (blue). The correlation coefficient of the second readout with its target function is nevertheless altogether 0.8032 ± 0.0087 and 0.8704 ± 0.0033 based on the actual state of the first readout, averaged over 50 simulation trials with 500 s of testing. The first readout represents an incorrect state for 5.75 ± 0.38% of the whole testing interval.
time (50 simulation trials with 500 s of testing). The additional readout unit, which computes the nonlinear function of the inputs $u_3(t)$ and $u_4(t)$, remains largely unaffected by the wrong state of the first readout (correlation coefficient: $0.9327 \pm 0.0022$ with its target function $f_3(t)$, average over 50 simulation trials with 500 s of testing).

**Analysis of Emerging Computational Mechanisms**

We analyzed the behavior of the network in the switchable routing task with the help of a PCA of the network activity vectors $r(t)$ during the test epoch. Figure 6A shows that the network dynamics resides in a higher-dimensional space than in the working memory task analyzed above. This is consistent with the demands of the task. Two effective dimensions are sufficient to keep the 2 items of the former task in working memory. In addition to those required for the 1-item memory, the switchable routing task demands dimensions for the routing of information, as well as for the nonlinear online processing task. The reward-modulated Hebbian learning rule thus autonomously adapts the dimensionality of the network dynamics to the task at hand (note that exactly the same network parameters were used for both simulations). This is also apparent in Figure 6B. Here, 2 attractor regions can be identified that are visited according to the actual activity of the memory readout. These regions, however, occupy a larger volume of the state space (compare with Fig. 4C), since

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**Figure 6.** Analysis of network dynamics in the switchable routing task. (A) Percentage of variance that can be explained by the first 1–10 principal components (#PCs) after learning. (B) First 3 principal components of the network trajectory after learning. The dynamics visits 2 subregions depending on the state of the memory readout (red: $z_1(t) < -0.3$; blue: $z_1(t) > 0.3$) and transitions between these regions (gray). (C) The histogram of correlation coefficients $cc_{Mem}$ between network neuron activities and the target function of the memory readout. (D) Neuron activity $r_1(t)$ of the neuron with maximal $cc_{Mem}$ (indicated by the black bar in panel C and arrow). The neuron is highly active whenever the memory readout should be active (indicated by gray shaded areas). (E) Pulse inputs $u_1$ (red) and $u_2$ (blue). (F) The histogram of correlation coefficients $cc_{u1}$ between network neuron activities and pulse input $u_1$. (G) Neuron activity $r_1(t)$ of the neuron with maximal $cc_{u1}$. The neuron exhibits peaks in its activity when a pulse in $u_1$ appears (compare with panel E). (H) The histogram of correlation coefficients $cc_{switched u3}$ between network neuron activities and the target function of the routing readout at times when analog input $u_3$ should be routed to the readout. (I) Neuron activity $r_1(t)$ of a neuron with large $cc_{switched u3}$ (black trace) along with analog inputs $u_3$ (red) and $u_4$ (blue). The neuron follows $u_3$ (red trace) when $u_3$ should be routed to the routing readout (indicated by gray shaded areas) and is preferentially highly active at other times.
additional computations have to be performed within the subspaces. Such attracting subspaces were termed high-dimensional attractors in Maass et al. (2007).

To further elucidate the computational mechanisms that emerged autonomously by reward-modulated Hebbian learning of readout weights, we analyzed the dynamics of neurons in the recurrent network after learning. Figure 6C shows the histogram of correlation coefficients \( c_{cMem} \) between network neurons' activities and the target function for readout \( z_1 \) (the memory readout). As exemplified in panel D, neurons with large \( c_{cMem} \) showed sustained activity during "on" states. These neurons had on average strong excitatory recurrent weights to each other (Supplementary Fig. 5A). They also had on average strong inhibitory recurrent weights to neurons that were anticorrelated with the memory target (Supplementary Fig. 5B). Interestingly, we observed no correlation between \( c_{cMem} \) of a network neuron and the weight of its projection to the memory readout. However, when we considered only neurons with \( c_{cMem} > 0.6 \), a positive correlation was significant (Supplementary Fig. 5C). Additionally, neurons with large \( c_{cMem} \) tended to receive strong feedback from the readout (Supplementary Fig. 5D). These findings indicate that the working memory benefitted from recurrent connections in the network. The readout utilized such network neurons, but in a complex way. Successively reducing the feedback weights from the readout during the test interval leads to an amplification of the readout output, rather than to a reduction (Supplementary Fig. 6). This indicates a regulatory role of the readout, rather than just a boosting of activity through positive feedback.

How did the pulse input \( u_4 \) influence the working memory? Neurons with large \( c_{cMem} \) had on average no preference for input channel \( u_1 \) which initiates the persistent activity of readout \( z_1 \). Instead, we found a number of neurons in the network that were correlated with the pulse input \( u_1 \); see Figure 6E (and analogous neurons for \( u_2 \), not shown). These correlations \( c_{c1u1} \) were not particularly strong. But as shown in Figure 6G for the most strongly correlated neuron, these neurons had peak activities whenever \( u_1 \) was active (compare with panel E), resulting from strong synaptic inputs from \( u_1 \) (Supplementary Fig. 5E). The direct influence of such neurons on neurons with large \( c_{cMem} \) was presumably weak, as their weights to these neurons were small on average (Supplementary Fig. 5F). We thus hypothesized that the readout utilized these neurons to switch itself and therefore the whole attractor. Surprisingly, we found only a slight correlation between \( c_{c1u1} \) of a network neuron and its weight to the readout neuron (Supplementary Fig. 5G). To further elaborate on this point, we performed control simulations where we set all weights in the recurrent network to zero. We found that no attractor was attained. Instead, the memory readout mimicked \( u_1 \) and \( u_2 \). This shows that some disynaptic connections—from the switching inputs to the network, and then directly to the readout—substantially participated in switching the readout.

Finally, we investigated the question of how the network achieved switchable routing. We found neurons that were significantly correlated with the target function of the routing readout (maximal \( c_{crouting} = 0.67 \)). These neurons were utilized by readout \( z_2 \) (the routing readout), since such neurons had preferentially strong projections to it (Supplementary Fig. 5H). Additionally, a number of neuron activities had an even higher correlation coefficient \( c_{c_{switched} u_3} \) with the target of the switchable routing task if only times were considered when \( u_3 \) should be routed to the output (Fig. 6H). Similar neurons were found for \( u_4 \). An example neuron with large \( c_{c_{switched} u_3} \) is shown in Figure 6I. It is also evident that this neuron was not correlated with \( u_4 \) (blue trace). As expected, neurons with large \( c_{c_{switched} u_3} \) received strong input from \( u_3 \) (Supplementary Fig. 5I). Such neurons, which follow the desired analog signal in the corresponding memory states and which are indifferent to the analog input in the other memory state, are presumably valuable presynaptic partners for the readout neuron. Consistent with this view, such neurons showed preferentially strong projections to the routing readout (Supplementary Fig. 5J).

In summary, this brief analysis shows that surprisingly complex computational mechanisms can be induced in a generic recurrent circuit by adapting the synaptic efficacies to readout neurons with a reward-modulated Hebbian learning rule.

**Discussion**

We have shown that heterogeneous and specialized computational functions can emerge through synaptic plasticity in generic sparsely connected recurrent networks of neurons. Furthermore, different computational structures can emerge simultaneously, even without an instructive teacher signal that tells each neuron what it should compute. Finally, we have shown that neural circuits need to receive very little information about the target output of each readout neuron: It suffices if they all receive a single global signal, which informs them whether their combined average performance has recently improved, or not. Arguably, this is the least informative performance-related signal that one can possibly conceive. We conjecture that if one reduces the information content of this global signal even further, then goal-directed learning is no longer possible.

A common feature of the 4 computational tasks considered in this paper is that there exists substantial evidence that neural networks of primates and other animals can carry out these tasks. Primates can learn to generate an immense variety of periodic movements, but it is not known how these capabilities are acquired and stored. Our results imply that, in principle, no specialized genetically encoded neural networks are required for this. This is of interest, because to the best of our knowledge no evidence for the existence of the latter has been found in primates. Furthermore, there exists a large body of experimental evidence that primates (Rodriguez and Paule 2009) and rodents (Rich and Shapiro 2009; Durstewitz et al. 2010) can learn rules for behaving that depend on specific cues, in a way that is likely to be rewarded. To accomplish such behavior, they must be able to keep relevant cues in working memory for extended periods of time. But it has remained an open problem how such rules are learned, and how working memory is implemented in neural networks of the brain. One remarkable recent experimental study (Bernacchia et al. 2011) suggests that working memory is implemented through heterogeneous neural subpopulations with different temporal responses, in a manner similar to that which emerges through reward-modulated learning in our study. A number of researchers have postulated that context- and task-dependent routing of information is required in the
brain for language understanding (Dominey et al. 2006), and for many other tasks where abstract knowledge needs to be applied for processing sensory input streams (Olshausen et al. 1993). Previous models for flexible routing of information in networks of neurons (Anderson and Van Essen 1987; Olshausen et al. 1995; von der Malsburg 2003; Zylberberg et al. 2010) required specially constructed neural networks. Recently, Vogels and Abbott (2009) proposed a model that is based on balanced excitation and inhibition. Additionally, several models have been introduced where synchronous activation is utilized for information routing in neural networks (Crick and Koch 1990; Salinas and Sejnowski 2001; Fries 2005; Akam and Kullmann 2010). Our work proposes a new model that relies neither on assumptions about synchronous network activity nor explicitly on balanced synaptic input. According to our model, generic networks of neurons can learn to route information as needed for specific computational tasks. Finally, it had already been shown in Maass et al. (2002) that specific analog computations on complex input signals can be learned by the readouts from a generic recurrent network of neurons through supervised learning. We show here that this learning also succeeds with a biologically more realistic feedback signal that does not require a teacher.

Some specific conditions on the structure and dynamics of networks of neurons have to be met in order to make this learning result possible. One is that the output of neural circuits needs to exhibit trial-to-trial variability (i.e., stochasticity). Without that, no exploration of possibly better network specialization is possible. Another condition is that neural circuits need to have some basic features, which enable them to acquire virtually arbitrary computational specializations by just changing the synaptic weights of a few neurons. The underlying theory (Maass et al. 2007) guarantees only that this can be achieved if readout neurons can compute arbitrary continuous functions (without memory). However, if the neural circuit provides sufficiently rich generic nonlinear preprocessing (see the kernel property of Legenstein and Maass 2007), linear readouts tend to suffice (Maass et al. 2002; Legenstein and Maass 2007). Finally, for some of the desired computations (those that require a nonfading memory), it is necessary that readout neurons whose synaptic inputs are subject to synaptic plasticity also project their output back into the circuit.

In this paper, we considered only rate models for generic cortical microcircuits. Similar network models have been previously used to model the dynamics of recurrent biological networks of neurons (Amari 1972; Hopfield 1984; Haykin 1999; Sussillo and Abbott 2009). It has been shown in computer simulations that some related computational tasks can be learned by networks of integrate-and-fire neurons through supervised learning (Maass et al. 2007). This indicates that the general set-up is in principle compatible with networks of spiking neurons, although the network size will have to be increased considerably for comparable performance. Reward-modulated Hebbian learning rules similar to the EH rule used in this paper have also been formulated for spiking neurons (Fiete and Seung 2006). Such spike-based learning rules in this paper have also been formulated for spiking neurons modulated Hebbian learning rules similar to the EH rule used. Increased considerably for comparable performance. Reward-general set-up is in principle compatible with networks of neurons, and the release of dopamine from such projections has been related to the expression of synaptic plasticity. But how this could be implemented is a wide open question, as are most questions regarding the molecular mechanisms that implement synaptic plasticity, and their exact time courses.

**Experimentally Testable Predictions of Our Model**

A primary prediction of our model is that no drastic differences in the structure of cortical microcircuits that perform different types of computations (periodic pattern generations, working memory, switchable routing of information, and nonlinear online computations) should be expected. Furthermore, our model predicts that a large number of different brain areas can learn to carry out these computations. In fact, several studies indicate that network function is only partially genetically predetermined, for example, in networks that generate periodic patterns for locomotion (Marder and Goaillard 2006) and in the auditory cortex (von Melchner et al. 2000). Other predictions of our model concern the organization of learning, more precisely the information provided by modulatory signals that gate synaptic plasticity.

Many studies indicate that the frontal cortex contains neurons that are sensitive to errors (Ridderinkhof et al. 2004) as well as neurons that track past or future performance (Hasegawa et al. 2000). In traditional error-based learning approaches involving multiple readouts, each readout unit was supervised individually by providing it with its exact error (or even with its target value). According to our model, it is not necessary that an individual modulatory signal is provided to each readout. Instead, we use the same modulatory signal for all readouts, indicating only whether the collective performance has increased due to random noise perturbations. Consider, for example, a task where the overall error is given by the combined error in several subtasks. Our model suggests that such tasks can in principle be learned, even if only the overall error but not the individual error signals can be extracted from the sensory information. The fact that animals are able to learn motor tasks that demand the coordination of many muscle activations or motor synergies indicates that this is indeed possible.

Movement-related rhythmic activity patterns related to jaw and tongue movements have been found in the primary motor cortex of primates (Yao et al. 2002), which has been shown to be involved in learning of fine motor skills (Molina-Luna et al. 2009). Traces of persistent memory, reflected in sustained firing activity of single neurons in response to specific visual stimuli, have been recorded in the prefrontal cortex (Fuster and Alexander 1971; Goldman-Rakic 1995; Miller et al. 1996). Primary motor cortex and prefrontal cortex receive input projections from midbrain dopaminergic neurons, and the release of dopamine from such projections has been related to the expression of synaptic plasticity in this area (Molina-Luna et al. 2009; Hosp et al. 2011). Therefore, the entrainment of such movement-related activity patterns in
the primary motor cortex and persistent memory traces in the prefrontal cortex may also be guided by modulatory input from midbrain neuromodulatory signals. Assuming that the proposed tasks are performed using such a learning mechanism, our model predicts that the synaptic adaptations that keep the desired trajectories stable during learning depend on the availability of global signals, such as specific neuromodulators. Without the presence of such signals, adaptation would not be possible. This is consistent with studies showing that both working memory performance and motor skill learning are impaired if the dopaminergic system of the brain is degenerated, as in patients with Parkinson's disease (Durstewitz and Seamans 2002; Doyon 2008; Molina-Luna et al. 2009), and also with studies showing that working memory performance is impaired if dopaminergic input to the prefrontal cortex is blocked (Durstewitz and Seamans 2002). Moreover, our simulation results are also consistent with results indicating that dopaminergic signaling in the primary motor cortex is involved in learning new motor skills, but not in executing a previously learned skill (Molina-Luna et al. 2009; Hosp et al. 2011).

Conclusion
In summary, we have shown how diverse computational functions, such as periodic pattern generation, memory-dependent computations, and state-dependent routing of information, could be attained and maintained by generic cortical microcircuits, such as periodic pattern generation, memory-dependent computations, and state-dependent routing of information, could be attained and maintained by generic cortical microcircuits via a biologically plausible 3-factor learning rule. It suffices that local synaptic learning rules receive a global modulatory signal that transmits some minimal information about global changes in task performance. Our results suggest that neuronal variability plays a crucial role in this learning process. It enables generic networks of neurons to learn important computational tasks without any supervisor or teacher (as postulated in previous work on the liquid computing model), simply through trial and error.

Supplementary Material
Supplementary material can be found at: http://www.cercor.oxfordjournals.org/

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